Errata and Comments for "Energy of knots and conformal geometry"

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Abstract

This article serves as errata of the book

"*Energy of knots and conformal geometry*", Series on Knots and Everthing Vol. 33, World Scientific, Singapore, 304 pages, (2003).

The updated version is available through web, linked from the author's Home Page:

http://www.comp.tmu.ac.jp/knotNRG/indices/indexe.html

The list below would be far from being completed[1].
Suggestions and comments would be deeply appreciated.

Key words and phrases. Energy, knot, link, Möbius geometry, conformal geometry, cross ratio
1991 *Mathematics Subject Classification.* Primary 57M25, Secondary 53A30

- Throughout the book ${}_nC_p$ means the number of combination $\binom{n}{p} = \frac{n!}{p!(n-p)!}$. The author is sorry for the Japanese notation.

- Page 3. The second paragraph

  "Let M denote $\mathbb{R}^3$ or S^3. Two knots f and f' in M are called **isotopic** if there is an isotopy $h_t : M \to M$ ($t \in [0,1]$) of the ambient space such that h_0 is equal to the identity map and that the map $(x,t) \mapsto (h_t(x),t)$ from $M \times [0,1]$ to itself is a homeomorphism. Then two knots are isotopic if and only if there is an orientation preserving homeomorphism h of M that satisfies $f' = h \circ f$. A **knot type** $[K]$ a knot K is an isotopy class of K. "

  should be replaced by

  "Let M denote $\mathbb{R}^3$ or S^3. Two knots f and f' in M are called **isotopic** if there is an isotopy $h_t : M \to M$ ($t \in [0,1]$) of the ambient space such that h_0 is equal to the identity map, that the map $(x,t) \mapsto (h_t(x),t)$ from $M \times [0,1]$ to itself is a homeomorphism, **and that $\boldsymbol{h_1 \circ f = f'}$**. Then two knots are isotopic if and only if there is an orientation preserving homeomorphism h of M that satisfies $f' = h \circ f$. **In this book, let us call an isotopy class of a knot K a knot type of** $[K]$**.** "

- Page 15. The second line from the bottom.

  $$E_\varepsilon^{(\alpha)}(K) = E_\varepsilon^{(\alpha)}(h) = \int_{\boldsymbol{S^1}} V_\varepsilon^{(\alpha)}(K;x)dx = \int_{S^1} V_\varepsilon^{(\alpha)}(h;s)ds.$$

  should be replaced by

  $$E_\varepsilon^{(\alpha)}(K) = E_\varepsilon^{(\alpha)}(h) = \int_{\boldsymbol{K}} V_\varepsilon^{(\alpha)}(K;x)dx = \int_{S^1} V_\varepsilon^{(\alpha)}(h;s)ds.$$

- Page 26. "then $E^{(\alpha)}$" in the first line of Theorem 2.4.1 (2) should be removed.

[1]The author can find errors almost every time he has a look at the book.

- Page 26. The 5th line from the bottom, i.e. the second line in the proof of Theorem 2.4.1 (2).
  "Let h be a knot with $|\boldsymbol{h''}| \equiv \mathbf{1}$ and $b = E^{(\alpha)}(h)$ $(b > 0)$."
  should be replaced by
  "Let h be a knot with $|\boldsymbol{h'}| \equiv \mathbf{1}$ and $b = E^{(\alpha)}(h)$ $(b > 0)$."
- Page 34. **Definition 3.1**. The first line of the formula holds if $X \neq P$:

$$I_\Sigma(X) = \begin{cases} P + \dfrac{r^2}{|X-P|^2}(X-P) & \text{if} \quad X \in \mathbb{R}^3, \\ \infty & \text{if} \quad X = P, \\ P & \text{if} \quad X = \infty. \end{cases}$$

  should be replaced by

$$I_\Sigma(X) = \begin{cases} P + \dfrac{r^2}{|X-P|^2}(X-P) & \text{if} \quad X \in \mathbb{R}^3 \backslash \boldsymbol{\{P\}}, \\ \infty & \text{if} \quad X = P, \\ P & \text{if} \quad X = \infty. \end{cases}$$

- Page 42. Outline of proof of Theorem 3.5.1.
  The convergence means the convergence with respect to the C^0-topology.
- Page 43. The first line. "Kusner and J. Sullivan" should be replaced by "Kim and Kusner".
- The book does not contain the proofs of Theorem 3.6.1 on page 44 and Theorem 3.7.7 on page 53, for which the reader is referred to [FHW] and [He1] respectively.
- Page 45. The right hand side of the 6th line from the bottom.

$$-\varepsilon \left[(1-\xi)\left\{|\boldsymbol{f'}|'(s+\varepsilon\xi)\right\}\right]_0^1 + \varepsilon^2 \int_0^1 (1-\xi)\left\{|f'|'(s+\varepsilon\xi)\right\} d\xi$$

  should be replaced by

$$-\varepsilon \left[(1-\xi)\left\{|\boldsymbol{f'}|(s+\varepsilon\xi)\right\}\right]_0^1 + \varepsilon^2 \int_0^1 (1-\xi)\left\{|f'|'(s+\varepsilon\xi)\right\} d\xi$$

- Page 54. Section 3.8.
  The main point of [KK] is as follows: As r approaches 0 or 1, the orbital configurations approach a p- or q-fold covered circle, and thus its energy $E = E^{(2)}$ must go to infinity. Since E is continuous, it takes on a minimum at some r_0, and by the symmetric criticality argument, this is actually critical among all variations, not just the orbital ones.
- Page 57. The 4th line from the bottom.

$$\int_{|z|=1} \left(\frac{i(r^2p^2 + (1-r^2)q^2)}{r^2 z^{1-p}(z^p-1)^2 + (1-r^2)z^{1-q}(z^q-1)^2} - \frac{i}{(z-1)^2} \right) \boldsymbol{ds}$$

  should be replaced by

$$\int_{|z|=1} \left(\frac{i(r^2p^2 + (1-r^2)q^2)}{r^2 z^{1-p}(z^p-1)^2 + (1-r^2)z^{1-q}(z^q-1)^2} - \frac{i}{(z-1)^2} \right) \boldsymbol{dz}$$

- Page 58, The 8th line in subsection 3.9.1.

$$\begin{aligned} \mathrm{Conf}_{n,m}(K, \boldsymbol{S^3}) &= \left\{(x_1, \cdots, x_{n+m}) \in K^n \times (\boldsymbol{S^3})^m \,\middle|\, x_j \neq x_k \, (j \neq k)\right\}, \\ U_D &= \left\{(x_1, \cdots, x_{n+m}) \in \mathrm{Conf}_{n,m}(K, \boldsymbol{S^3}) \,\middle|\, x_1 \prec \cdots \prec x_n\right\}. \end{aligned}$$

  should be replaced by

$$\begin{aligned} \mathrm{Conf}_{n,m}(K, \mathbb{R}^3) &= \left\{(x_1, \cdots, x_{n+m}) \in K^n \times (\mathbb{R}^3)^m \,\middle|\, x_j \neq x_k \, (j \neq k)\right\}, \\ U_D &= \left\{(x_1, \cdots, x_{n+m}) \in \mathrm{Conf}_{n,m}(K, \mathbb{R}^3) \,\middle|\, x_1 \prec \cdots \prec x_n\right\}. \end{aligned},$$

  i.e. S^3 should be replaced by $\mathbb{R}^3$, as we use Euclidean metric in this subsection.

- Page 59, in Definition 3.5.

$$E_{X,\cos}(K) = \int_{K^4;\, x_1 \prec \boldsymbol{x_3} \prec \boldsymbol{x_2} \prec x_4} \frac{(1-\cos\theta_{13})(1-\cos\theta_{24})}{|x_1-x_3|^2|x_2-x_4|^2}\, dx_1dx_2dx_3dx_4,$$
$$E_{X,\sin}(K) = \int_{K^4;\, x_1 \prec \boldsymbol{x_3} \prec \boldsymbol{x_2} \prec x_4} \frac{\sin\theta_{13}\sin\theta_{24}}{|x_1-x_3|^2|x_2-x_4|^2}\, dx_1dx_2dx_3dx_4,$$

should be replaced by

$$E_{X,\cos}(K) = \int_{K^4;\, x_1 \prec \boldsymbol{x_2} \prec \boldsymbol{x_3} \prec x_4} \frac{(1-\cos\theta_{13})(1-\cos\theta_{24})}{|x_1-x_3|^2|x_2-x_4|^2}\, dx_1dx_2dx_3dx_4,$$
$$E_{X,\sin}(K) = \int_{K^4;\, x_1 \prec \boldsymbol{x_2} \prec \boldsymbol{x_3} \prec x_4} \frac{\sin\theta_{13}\sin\theta_{24}}{|x_1-x_3|^2|x_2-x_4|^2}\, dx_1dx_2dx_3dx_4,$$
,

i.e. the order of x_2 and x_3 in "$x_1 \prec x_3 \prec x_2 \prec x_4$" should be reversed.

- Page 67. The 5th line

  If $E^{\alpha,p}(h)$ with $\alpha p > 2$ is **finite** then h cannot have a sharp turn.

  should be replaced by

  If $E^{\alpha,p}(h)$ with $\alpha p > 2$ is **bounded** then h cannot have a sharp turn.

- Pages 95, Figure 6.7, page 97, Figure 6.11, page 99, Figure 6.15. The caption "Look with the **right** eye." should be "Look with the **left** eye." (You do not have to close your left eye.)

- There is a misunderstanding about the order of contact. The order of contact in the book should be reduced by 1. The errors can be found on pages 119, 120, 123-125, 184-185.

  - Page 119. Definition 8.2

    "(1) An **osculating circle** $\cdots$ is the circle which is tangent to K at x at least to the **third** order"

    should be replaced by

    "(1) An **osculating circle** $\cdots$ is the circle which is tangent to K at x at least to the **second** order"
    and
    "(2) An **osculating sphere** $\cdots$ is tangent to K at x at least to the **fourth** order. "
    should be replaced by
    "(2) An **osculating sphere** $\cdots$ is tangent to K at x at least to the **third** order. "

  - Page 119.

    "**Proposition 8.3.1**(1) *An osculating sphere is uniquely determined if the order of tangency of the osculating circle to the knot is just* **3**.

    (2) *Suppose the order of tangency of the osculating circle to the knot is just* **3**. *Then* $\cdots$ "

    should be replaced by

    "**Proposition 8.3.1**(1) *An osculating sphere is uniquely determined if the order of tangency of the osculating circle to the knot is just* **2**.

    (2) *Suppose the order of tangency of the osculating circle to the knot is just* **2**. *Then* $\cdots$ "

  - Page 120. After Proposition 8.3.1.

    "When the order of tangency of the osculating circle to the knot is greater than **3**, any 2-sphere through the osculating circle is tangent to the knot to the **fourth** order. But there might be a unique 2-sphere which is tangent to the knot with a higher order of tangency than **4**. "

    should be replaced by

    "When the order of tangency of the osculating circle to the knot is greater than **2**, any 2-sphere through the osculating circle is tangent to the knot to the **third** order. But there might be a unique 2-sphere which is tangent to the knot with a higher order of tangency than **3**. "

- Page 120. The 6th line from the bottom (after the formula (8.1)).

  "Then C is tangent to K at 0 to the **fourth** order, i.e. $f^{(3)}(0) = C^{(3)}(0)$, if and only if $k' = 0$ and $k\tau = 0$. "

  should be replaced by

  "Then C is tangent to K at 0 to the **third** order, i.e. $f^{(3)}(0) = C^{(3)}(0)$, if and only if $k' = 0$ and $k\tau = 0$. "

- Pages 123-124. Proof of Proposition 8.4.2.

  "Case I: $\cdots$

  If the knot K is transversal to Σ at x and if the order of tangency of K and Σ at y is **2**, then K must intersect Σ (not necessarily transversally) at a third point z which is different from both x and y, and therefore $\Sigma = \sigma(x, z, y, y)$.

  If the order of tangency of K and Σ at y is more than or equal to **3**, then $\Sigma = \sigma(x, y, y, y)$.

  Case II: Suppose $x = y$. The order of tangency of K to Σ at x is more than or equal to **3**.

  If it is **3** then K must intersect Σ (not necessarily transversally) at another point z ($z \neq x$). Then $\Sigma = \sigma(x, x, x, z)$.

  If the order of tangency is more than or equal to **4** then $\Sigma = \sigma(x, x, x, x)$. "

  should be replaced by

  "Case I: $\cdots$

  If the knot K is transversal to Σ at x and if the order of contact of K and Σ at y is **1**, then K must intersect Σ (not necessarily transversally) at a third point z which is different from both x and y, and therefore $\Sigma = \sigma(x, z, y, y)$.

  If the order of contact of K and Σ at y is more than or equal to **2**, then $\Sigma = \sigma(x, y, y, y)$.

  Case II: Suppose $x = y$. The order of tangency of K to Σ at x is more than or equal to **2**.

  If it is **3** then K must intersect Σ (not necessarily transversally) at another point z ($z \neq x$). Then $\Sigma = \sigma(x, x, x, z)$.

  If the order of contact is more than or equal to **3** then $\Sigma = \sigma(x, x, x, x)$. "

- Page 125. The 1st line.

  "Since $\rho(x) > r(\Sigma)$ by the assumption, the knot K cannot have the tangency of order **3** with Σ at x, and hence K must lie in $\cdots$"

  should be replaced by

  "Since $\rho(x) > r(\Sigma)$ by the assumption, the knot K cannot have the tangency of order **2** with Σ at x, and hence K must lie in $\cdots$"

• Page 125. The 5th line.

  "then the osculating sphere at x contains the osculating **sphere** at x as the great circle"

  should be replaced by

  "then the osculating sphere at x contains the osculating **circle** at x as the great circle"

• Page 141. The 7th and 8th lines from the bottom, i.e. the last two lines of (4) have two errors:

  Since $u_5\_v_5 > 1$ this implies $u_1v_1 + \cdots + u_4v_4 \le u_5v_5 - 1$ which means $\langle \boldsymbol{u}, \boldsymbol{v} \rangle < -1$.

  should be replaced by

  Since $u_5v_5 > 1$ this implies $u_1v_1 + \cdots + u_4v_4 \le u_5v_5 - 1$ which means $\langle \boldsymbol{u}, \boldsymbol{v} \rangle \le -1$.

  Furthormore, the proof that $\langle \boldsymbol{u}, \boldsymbol{v} \rangle \neq -1$ should be added:

  Assume $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = -1$. Since $\langle \boldsymbol{u}, \boldsymbol{u} \rangle = -1$ we have $\langle \boldsymbol{u}, \boldsymbol{u} - \boldsymbol{v} \rangle = 0$. Lemma 9.1.1 (3) implies that $\boldsymbol{u} - \boldsymbol{v}$ is either space-like or equal to $\boldsymbol{0}$. As $\langle \boldsymbol{u} - \boldsymbol{v}, \boldsymbol{u} - \boldsymbol{v} \rangle = 0$, $\boldsymbol{u} - \boldsymbol{v}$ cannot be space-like. Therefore, $\boldsymbol{u} - \boldsymbol{v} = \boldsymbol{0}$, which is a contradiction.

• Page 145. The 3rd line. S^3_{infty} should be replaced by S^3_∞.

• Page 145. The bijection in Theorem 9.3.2 can be considered as a modern version of the pentaspherical coordinates in [Dar].

- Page 151 last two lines to Page 152 line. (1a). The book misses the description of the case when $n = 1$.

  When $n = 1$ $P^{\perp} \cap S^1_{\infty} = \emptyset$ since $P^{\perp}$ is a time-like line. In this case we may consider the "base sphere" to be $\emptyset = \partial(P^{\perp} \cap \mathbb{H}^2)$, where $\partial \mathbb{H}^2 = S^1_{\infty}$. Let us call $\mathcal{P}$ or the set of corresponding spheres of dimension 0 a "*space-like pencil*".

- Page 166. Three lines before **Lemma 9.8.2**.

$$\psi : \mathbb{R}^{n+1}_{+} \ni (\boldsymbol{X}, r) {=} \varphi^{-1} \circ p^{-1} \left(S^{n-1}_{r}(\boldsymbol{X}) \right) \in \Lambda \setminus (\mathrm{Span}\,\langle Q \rangle)^{\perp}$$

  should be replaced by

$$\psi : \mathbb{R}^{n+1}_{+} \ni (\boldsymbol{X}, r) {\mapsto} \varphi^{-1} \circ p^{-1} \left(S^{n-1}_{r}(\boldsymbol{X}) \right) \in \Lambda \setminus (\mathrm{Span}\,\langle Q \rangle)^{\perp}.$$

- Page 167. The 8th line.

$$\omega_{\mathbb{R}^{n+1}_{+}} = \frac{1}{\boldsymbol{x_{n+2}}}\, dx_1 \wedge \cdots \wedge dx_{n+1}$$

  should be replaced by

$$\omega_{\mathbb{R}^{n+1}_{+}} = \boldsymbol{-}\frac{1}{\boldsymbol{r^{n+1}}} dX_1 \wedge \cdots \wedge dX_n \boldsymbol{\wedge dr}$$

- Page 167. The 16th line.

$$\widetilde{g} : \mathbb{R}^{n+1}_{+} \ni (\boldsymbol{X}, r) \mapsto \left(\frac{\boldsymbol{X}}{|\boldsymbol{X}|^2 - r^2}, \frac{r}{|\boldsymbol{X}|^2 - r^2} \right) \in \mathbb{R}^{n+1}_{+}.$$

  should be replaced by

$$\widetilde{g} : \mathbb{R}^{n+1}_{+} \ni (\boldsymbol{X}, r) \mapsto \left(\frac{\boldsymbol{X}}{|\boldsymbol{X}|^2 - r^2}, \frac{r}{||\boldsymbol{X}|^2 - r^2|} \right) \in \mathbb{R}^{n+1}_{+}.$$

- Page 175. **Definition 10.1** (2) should be replaced by

  "(2) An oriented 2-sphere Σ is called a **non-trivial sphere in the strict sense** for a knot K if each connected component of $\mathbb{R}^3 \setminus \Sigma$ ($S^3 \setminus \Sigma$) contains at least 2 connected components of $K \setminus (K \cap \Sigma)$. "

- Page 184. **Definition 10.6**. (1)

$$\mathcal{C}c^{(4)}(K) = \left\{ (s,s,s,s) \in \Delta^{(4)} \,\middle|\, \begin{array}{l} \text{The osculating circle of } K \text{ at } f(s) \\ \text{is tangent to } K \text{ to the } \textbf{fourth} \text{ order} \end{array} \right\}.$$

  should be replaced by

$$\mathcal{C}c^{(4)}(K) = \left\{ (s,s,s,s) \in \Delta^{(4)} \,\middle|\, \begin{array}{l} \text{The osculating circle of } K \text{ at } f(s) \\ \text{is tangent to } K \text{ to the } \textbf{third} \text{ order} \end{array} \right\}.$$

- Page 184. **Proposition 10.3.3**. (1)

  "The order of tangency of the osculating circle at $f(s)$ is exactly **3** if and only if $f(s) \wedge f'(s) \wedge f''(s) \wedge f'''(s) \neq \mathbf{0}$. "

  should be replaced by

  "The order of tangency of the osculating circle at $f(s)$ is exactly **2** if and only if $f(s) \wedge f'(s) \wedge f''(s) \wedge f'''(s) \neq \mathbf{0}$. "

- Page 185. The 6th line and the 10th line from the bottom (in the *Proof* of Proposition 10.3.3 (1)).

  "(1) Suppose the osculating circle C is tangent to the knot at $f(s)$ to the **fourth** order. $\cdots$

  $\cdots$ and hence the osculating circle C is tangent to the knot at $f(s)$ to the **fourth** order. "

  should be replaced by

  "(1) Suppose the osculating circle C is tangent to the knot at $f(s)$ to the **third** order. $\cdots$

  $\cdots$ and hence the osculating circle C is tangent to the knot at $f(s)$ to the **third** order. "

- Page 191. The 7th line.

$$(I \times I)^* \lambda_{\Re} = \lambda_{\Re} + \frac{1}{2} d \log(|\boldsymbol{u}|^{\mathbf{2}})$$

should be replaced by

$$(I \times I)^* \lambda_{\Re} = \lambda_{\Re} + \frac{1}{2} d \log(\boldsymbol{u}^{\mathbf{2}} + \boldsymbol{v}^{\mathbf{2}})$$

- Page 193. The 4th line in the Proof of Lemma 11.2.2. '**bas**' should be replaced by '**basis**'.

- Page 193. The 5th line in Lemma 11.2.3. '**this** local coordinate' should be replaced by '**these** local coordinate**s**'.

- Page 198. Theorem 11.2.7. As a corollary of this theorem, we have

  Corollary The pull-back $\omega = \psi^* \omega_0$ of the canonical symplectic form ω_0 of T^*S^n by $\psi : S^n \times S^n \setminus \Delta \to T^*S^n$ is invariant under any diagonal action of the Möbius group $\mathcal{M}$ on $S^n \times S^n \setminus \Delta$.

- Page 200. The condition (ii) of Theorem 11.2.9 is not necessary.

- Page 203. A remark on Definition 11.4:

  The real part of the infinitesimal cross ratio of a knot K is a smooth 2-form on $K \times K \setminus \triangle$, but it is not the case with the imaginary part. Since the conformal angle $\theta_K(x, y)$ is not a smooth function of x and y (see Figure 10.1 on page 183), **the imaginary part of the infinitesimal cross ratio may have singularity** at a pair of points $(x, y) \in K \times K \setminus \triangle$ where the conformal angle $\theta_K(x, y)$ vanishes.

- Page 206. The 2nd and 3rd lines

$$\big((T_0 \circ f)^* dx_3\big)(s_0, t_0) = \left(\frac{d}{ds}(T_0 \circ f)_3\right)(s_0),$$

$$\big((T_0 \circ f)^* dy_3\big)(s_0, t_0) = \left(\frac{d}{dt}(T_0 \circ f)_3\right)(t_0)$$

should be replaced by

$$\big((T_0 \circ f)^* dx_3\big)(s_0, t_0) = \left(\frac{d}{ds}(T_0 \circ f)_3\right)(s_0) \mathbf{= 0},$$

$$\big((T_0 \circ f)^* dy_3\big)(s_0, t_0) = \left(\frac{d}{dt}(T_0 \circ f)_3\right)(t_0) \mathbf{= 0}$$

i.e. $= 0$ should be added at the end of the both formulae.

- Page 211. The 2nd and 3rd lines

$$\begin{aligned} \tfrac{1}{2} E_{\circ}^{(2)}(L_r) = E_{\circ}^{(2),mut}(L_r) &= \int_0^{2\pi}\int_0^{2\pi} \frac{ds dt}{2\mathbf{+2}\cos(t-s)} \\ &= 4\pi \int_0^{\frac{\pi}{2}} \frac{d\xi}{1 - r\cos 2\xi} \qquad (\boldsymbol{\xi} \mathbf{= \tan} \, \boldsymbol{\eta}) \end{aligned}$$

should be replaced by

$$\begin{aligned} \tfrac{1}{2} E_{\circ}^{(2)}(L_r) = E_{\circ}^{(2),mut}(L_r) &= \int_0^{2\pi}\int_0^{2\pi} \frac{ds dt}{2\mathbf{-2}\boldsymbol{r}\cos(t-s)} \\ &= 4\pi \int_0^{\frac{\pi}{2}} \frac{d\xi}{1 - r\cos 2\xi} \qquad (\boldsymbol{\eta} \mathbf{= \tan} \, \boldsymbol{\xi}) \end{aligned}$$

- Page 218. The 3rd line in the Proof of Lemma 12.3.2

$$V_{\sin\theta}(K; f(\pm t)), V_{\sin\theta}(K; f(\delta \pm t)) \boldsymbol{\geq} \frac{1}{100} \cdot \frac{1}{d+t}.$$

should be replaced by

$$V_{\sin\theta}(K; f(\pm t)), V_{\sin\theta}(K; f(\delta \pm t)) \boldsymbol{\leq} \frac{1}{100} \cdot \frac{1}{d+t}.$$

i.e. the inequality should be reversed.

- Page 218. The 8th line in the Proof of Lemma 12.3.2, i.e. just above Figure 12.2.

$$\lim_{K\setminus\{x_+\}\ni y\to x_+} \pi_{x_+}(\boldsymbol{y}) = \hat{x}_+.$$

should be replaced by

$$\lim_{K\setminus\{x_+\}\ni y\to x_+} \pi_{x_+}(\boldsymbol{I}_{\boldsymbol{x}_+}(\boldsymbol{y})) = \hat{x}_+.$$

- Page 218. The right picture of Figure 12.2 should be replaced by Figure 1. The radius of the circle, which

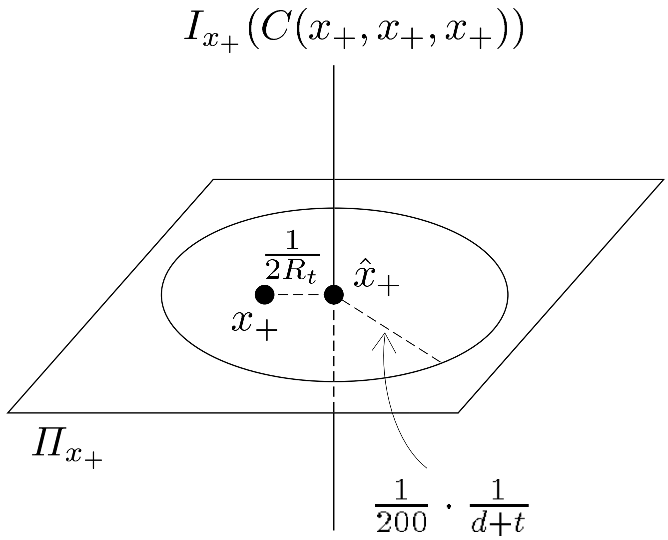

Figure 1: The right picture of Figure 12.2.

was $\frac{1}{400} \cdot \frac{1}{d+t}$ in the book, should be replaced by $\frac{1}{200} \cdot \frac{1}{d+t}$. Also, the point x_+ should be closer to the center of the circle, $\hat{x}_+$, as $|x_+ - \hat{x}_+| \le \frac{1}{400} \cdot \frac{1}{d+t}$, which is a half of the radius.

- Page 218. The 3rd line from the bottom.

  $\pi_{x_+}(\widetilde{K}_{x_+})$ lies inside the circle on Π_{x_+} with center $\boldsymbol{x}_+$ and radius $1/(200(d+t))$.

  should be replaced by

  $\pi_{x_+}(\widetilde{K}_{x_+})$ lies inside the circle on Π_{x_+} with center $\hat{\boldsymbol{x}}_+$ and radius $1/(200(d+t))$.

- Page 219. The 8th line (just above Figure 12.3).

  N_t whose meridian disc has radius $(\mathbf{400}(d+t))/3$, as illustrated in Figure 12.3.

  should be replaced by

  N_t whose meridian disc has radius $(\mathbf{200}(d+t))/3$, as illustrated in Figure 12.3.

- Page 219. Figure 12.3 should be replaced by Figure 2. Namely, the radius of the inner circle of the left picture, which was $\frac{1}{\mathbf{400}(d+t)}$ in the book, should be $\frac{1}{\mathbf{200}(d+t)}$, and the radius of the 'degenerate solid torus in the right picture, which was $\frac{\mathbf{400}(d+t)}{3}$ in the book, should be $\frac{\mathbf{200}(d+t)}{3}$.

- Page 219. The 7th line from the bottom

$$|f(t) - f(-t)| \le 2t \ll \frac{\mathbf{400}}{3}(d+t)$$

should be replaced by

$$|f(t) - f(-t)| \le 2t \ll \frac{\mathbf{200}}{3}(d+t)$$

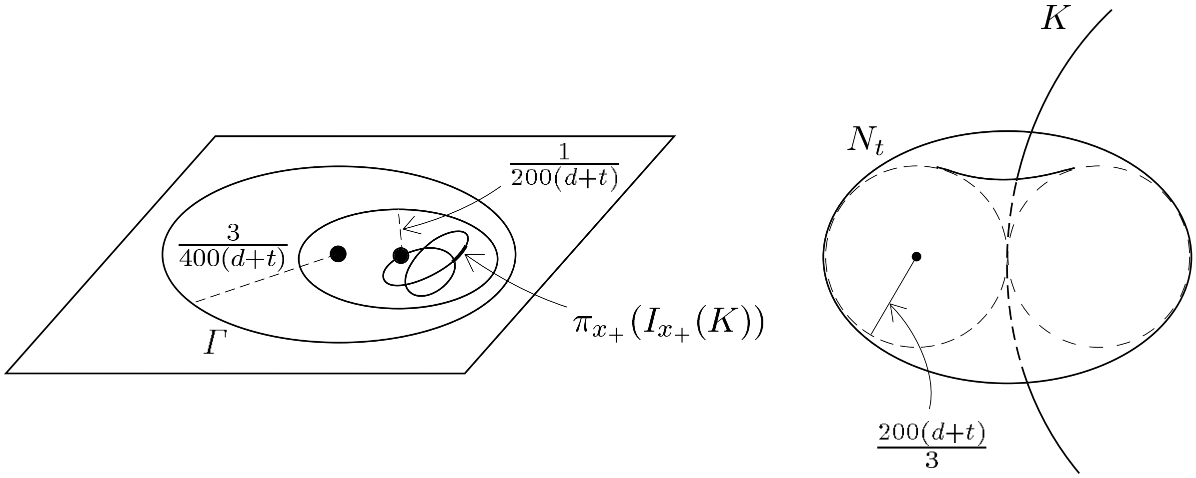

Figure 2: Figure 12.3.

- Pae 220. The 2nd line from the bottom

$$|f(0)-\boldsymbol{g(t)}| \geq \int_0^t (1-\kappa s)ds = t\left(1-\frac{\kappa}{2}t\right) > d,$$

should be replaced by

$$|f(0)-\boldsymbol{f(t)}| \geq \int_0^t (1-\kappa s)ds = t\left(1-\frac{\kappa}{2}t\right) > d,$$

- Page 226. The last three lines of Remark 13.2.2 should be replaced by
  This is because $1 \leq {}_nC_2 \leq {}_{2n}C_4$ when $n \geq 2$.
- Page 229. Line 6 up.
  "subarc of $C_3(r)$ between $Q_{12}(r)$ and $P_{\mathbf{23}}(r)$ "
  should be replaced by
  "subarc of $C_3(r)$ between $Q_{12}(r)$ and $P_{\mathbf{13}}(r)$ "
- Page 229. The last line. Page 230. The second line.
  The domain of integration "$\left[\frac{1}{2\sin\theta_K},\infty\right)$" should be replaced by "$\left[\frac{1}{2\sin\theta},\infty\right)$"
- Page 231. Lines 3-5. "$\cdots + o(|z|)$" should be replaced by "$\cdots + O(|z|)$".
- Page 236. The 13th line around the middle of the page

$$\int_{\Sigma_r(X,Y,Z)\cap K_d \neq \boldsymbol{\varphi}} \sharp(\Sigma_r(X,Y,Z)\cap K_d)\, dXdYdZ = 2\pi r^2\left(\bar{K}_d\right),$$

should be replaced by

$$\int_{\Sigma_r(X,Y,Z)\cap K_d \neq \boldsymbol{\emptyset}} \sharp(\Sigma_r(X,Y,Z)\cap K_d)\, dXdYdZ = 2\pi r^2\left(\bar{K}_d\right),$$

- Page 236. The 5th line from the bottom

$$E_{\mathrm{mnts}}(K_d) = \int_{\mathcal{S}(K_d)} C_2^{\sharp(\Sigma_r(X,Y,Z)\cap K_d)} \cdot \frac{1}{r^4}\, dXdYdZdr$$

should be replaced by

$$E_{\mathrm{mnts}}(K_d) = \int_{\mathcal{S}(K_d)} {}_{\frac{1}{2}\sharp(\Sigma_r(X,Y,Z)\cap K_d)}C_2 \cdot \frac{1}{r^4}\, dXdYdZdr$$

This implies the 4th line from the bottom because ${}_mC_2 \geq m-1$ for $m \geq 0$.

- Page 256. The 8th line (just before Example A.1) should be replaced by

  We show that $\boldsymbol{I_{tv}}$ can detect the unknot.

  Namely, it is a conjecture whether $|csl|$ can detect the unknot or not.

- Page 273.

  [CKS2] J. Cantarella, R. B. Kusner, and J. M. Sullivan, *On the minimum ropelength of knots and links,* Preprint.

  should be updated to

  [CKS2] J. Cantarella, R. B. Kusner, and J. M. Sullivan, *On the minimum ropelength of knots and links,* Invent. Math. **150** (2002), no. 2, 257–286.

  [CKS] should be removed as it is same as [CKS1].

- Page 276.

  [GLP] M. Gromov, J. Lafontaine, and P. **Pnau**, *Structures métriques pour les variétés riemanniennes,* Cedic/Fernand Nathan, Paris, 1981.

  should be replaced by

  [GLP]Gr-La-Pn M. Gromov, J. Lafontaine, and P. **Pansu**, *Structures métriques pour les variétés riemanniennes,* Cedic/Fernand Nathan, Paris, 1981.

- Page 279.

  [Lin] X.-S. Lin, *Knot energies and knot invariants,* **J. Differential Geom. 44 (1996), 74 − 95**.

  should be replaced by

  [Lin] X.-S. Lin, *Knot energies and knot invariants. Knot theory and its applications,* **Chaos Solitons Fractals 9 (1998) 645 − 655**

- Page 285, Index. The "average crossing number" also appears on page 44.

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